

Three-dimensional localized coherent structures of surface turbulence. III Experiment and model validation.

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Abstract

The paper continues a series of publications devoted to the 3D nonlinear localized coherent structures on the surface of vertically falling liquid films. The work is primarily focussed on experimental investigations. We study: (i) instabilities and transitions leading to 3D coherent structures; (ii) characteristics of these structures. Some non-stationary effects are also studied numerically. Our experimental results, as well as the results of other investigators, are in a good agreement with our theoretical and numerical predictions.

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1 Introduction

A Motivation

Localized nonlinear coherent structures play an important role in many phenomena connected with the chaotic motion of liquid, in particular, in some regimes of falling films. As we discussed in Part I ^[1], the analysis of experiments reveals the existence of several wave regimes, depending on the inlet Reynolds number $\langle Re \rangle$, $(0, \langle Re^{(1)} \rangle)$, $(\langle Re^{(1)} \rangle, \langle Re^{(2)} \rangle)$ and $(\langle Re^{(2)} \rangle, \langle Re^{(3)} \rangle)$. For water, $\langle Re^{(1)} \rangle \approx 3 - 5$, $\langle Re^{(2)} \rangle \approx 40 - 70$ and $\langle Re^{(3)} \rangle \approx 400$. In the first interval the instability is too weak for any manifestation to be seen and the surface is flat, in the second interval the waves are two-dimensional (2D) and in the third interval the surface is covered with three-dimensional (3D) localized coherent structures, Λ -solitons. These 3D solitons are stable and robust and are involved in continuous chaotic movement, interacting with each other as “quasi-particles”. This is the so-called regime of “surface turbulence” (ST). Note that describing ST in terms of the motion of “quasi-particles” results in a dramatic reduction in the number of degrees of freedom.

In our theoretical investigation ^[1, 2] we systematically apply the simplified system of equations with respect to local film thickness and flow rates in the direction of the flow and in the transverse direction, the Kapitsa-Shkadov system. For large Kapitsa number $\gamma = \sigma \rho^{-1} \nu^{-4/3} g^{-1/3}$ (where σ is surface tension, ρ and ν are the liquid density and viscosity and g is the acceleration of gravity) the flow is described by the universal parameter δ (the modified Reynolds number) and the results can be re-calculated for any liquid using the relation $Re = 7.51 \gamma^{3/11} \delta^{9/11}$. In particular, for water, $Re = 66.1 \delta^{9/11}$. As $Re \rightarrow 0$, the Kapitsa-Shkadov system collapses into the Kuramoto-Sivashinsky (KS) equation.

In Part I ^[1], we have examined theoretically the instability of 2D waves with respect to a disturbances in the transverse direction. We have also established the 2D–3D transition scenarios resulting in fully developed 3D waves whose complex interaction leads eventually to ST. In Part II ^[2], we considered 3D coherent structures of the ST as stationary running 3D solitary pulses, referred to as Λ -solitons. We have found these nonlinear solutions and have studied their linear stability. It has been found that for the universal parameter

in the range $\delta < 0.054$ the Λ -solitons are unstable with respect to the continuous part of spectrum ("radiation instability") and, at $\delta > 0.51$, Λ -solitons do not exist. Hence their region of existence/stability is $0.054 < \delta < 0.51$ which for water translates to $6 < Re < 38$.

Among numerous works devoted to the experimental investigation of wave dynamics in falling films, only works by Park and Nosoko [3] and Alekseenko et al. [4, 5] are devoted to transitions to the regime of Λ -solitons and to the properties of these structures. Note that for films on an inclined plane with a small inclination angle, 3D solitary waves have not been observed (see papers by Liu et al. [6]).

In the work [3] water was the working liquid. The authors used artificial perturbations in the transverse direction to destroy 2D solitary waves, the critical Reynolds number for the transition to 3D waves was evaluated as $\langle Re \rangle \approx 40$. At this Reynolds number, the most dangerous length of transverse perturbations was about 2 cm. In this work, it was stated that disintegration of 2D solitons leads to 3D solitary waves. The work contains many nice photographs of the wavy surface, but, unfortunately, quantitative comparison with theory is rather difficult: i) a strong influence from the wave interaction complicates the analysis of the data, ii) the authors were using the inlet Reynolds number $\langle Re \rangle$, while for the comparison we need the substrate Reynolds number Re . However, there is a qualitative correspondence between the experimental data and our theory.

As a matter of fact, there are serious difficulties with trying to single out an individual "particle" of ST and its parameters from experiments. The main such difficulties are: a) the "particles" are the result of a long and complex evolution downstream; b) in the ST regime, the "particles" are continuously interacting with each other and it is very difficult if not impossible to measure the parameters of an individual particle; sometimes it is difficult even to identify them in a randomly disturbed surface.

In the experiments of Alekseenko et al. [4, 5], the evolution of 3D localized signals was considered for substrate Reynolds numbers $Re = 1.25 - 4.7$ in an alcohol-water mixture with $\gamma = 404$ so that the interval of the universal modified Reynolds number is $\delta = 0.015 - 0.076$. In these studies, the authors excited the 3D waves by a short-duration

impact with a thin jet of working fluid in the upper part of the film flow. The major trends of 3D localized signals were presented and the existence of stationary running 3D solitary waves was proved. For the time being, only these experiments contain enough information to be compared with the theoretical prediction.

The main aim of the present work is an experimental investigation of some 3D phenomena in vertically falling films and a validation of the model developed in Parts I and II. The range of Reynolds numbers $\langle Re \rangle$ we wish to investigate is 5 to 100 (smaller Reynolds numbers were investigated by Alekseenko et al.^[4, 5]). First, we study the 2D–3D transitions using method ^[3] and perturbing 2D solitary waves in the transverse direction. Second, we artificially create 3D solitary waves by depositing a single drop on the undisturbed liquid surface. A critical aspect of the experiment is that, in contrast to the works ^[4, 5], the mass of the deposited drop corresponds to the mass of 3D solitary wave obtained theoretically in Part II. To complete the picture of evolution, some nonstationary effects are studied numerically. The results of our experiments along with the experiments of Alekseenko et al. ^[4, 5] are compared with our theory in Parts I and II.

B Experimental methodology

We are going to investigate the 2D-3D transition and Λ -solitons for inlet Reynolds numbers $\langle Re \rangle$ from 5 to 100. For this purpose, we choose a planar vertical channel which is 25 cm long and 15 cm wide and made of a special mirrored glass. To measure the wave characteristics, we choose the fluorescent imaging method by Liu and Gollub ^[7]. The method was also successfully used by Vlachogiannis and Bontozoglou ^[8].

In order to investigate the transition from the regime of 2D solitary waves to Λ -solitons (2D-3D transition), we applied the ideas of ^[9, 10] and ^[3]: regular 2D solitary waves are created by low-frequency flow rate pulsations; these waves are perturbed in the transverse direction by a system of special needles separated by a certain distance. The spatio-temporal evolution of such waves are considered in order to find conditions of 3D instability and transition to 3D solitary waves.

We devise a novel direct method that allows us to create individual Λ -structures in

our experiments. The basic idea is to deposit drops having the soliton’s mass onto the liquid substrate. The mass of the solitary wave is taken from our theoretical calculations [2]. After the drop touches the surface, it is subsequently involved in film movement and is rapidly transformed into a stationary running solitary wave.

C Outline

In Sec. 2, we obtain the connection between the inlet or pump Reynolds number, $\langle Re \rangle$, frequently used in experiments, and the Reynolds number based on the soliton’s substrate, Re , used in our theoretical investigation [1]. The ratio $\langle Re \rangle / Re$ depends on the properties of the (3D or 2D) solitary wave, namely its volume and velocity, as well as on the “density” of solitons or, equivalently, the number of solitons per unit area. In the regime of ST, this ratio is about 10.

In Sec. 3, we describe our simple experimental setup and a method to measure wave characteristics. The planar vertical channel is made of a special mirrored glass and distilled water is used as working liquid. Experimental methods to investigate transitions leading to 3D solitary waves and to create regular 3D waves on the film surface are discussed. The numerical method used to study the evolution of localized signals is formulated.

In Sec. 4, we present our experimental results and compare them with the theoretical ones. First, some visual observations of natural waves and of their transitions are presented. Second, normal 2D solitary waves, created by periodic pulsations of flow rate, are considered. They are excited by special needles in the transverse direction. The evolution downstream of these perturbed 2D solitary waves can result in their disintegration and further formation of 3D solitary waves. The most dangerous wave length is found. Third, in order to create an individual 3D soliton, a drop of liquid with a mass corresponding to the equilibrium soliton is deposited directly onto the liquid substrate. Downstream, the drop spreads out and takes the form of a stationary running Λ -soliton. We also perform computations to model the creation of the Λ solitary wave from a localized initial signal, or “drop”. Good agreement between the theory in Parts I and II and our experiments

along with Alekseenko et al. [4, 5] is demonstrated. Finally, we consider a numerical solution of simple wave interactions.

Sec. 5 contains the main conclusions of the work and a discussion of unsolved problems for future investigation.

2 Connection between inlet and substrate Reynolds numbers

The natural control parameter in experiments is the pump discharge flow rate. The associated dimensionless parameter is the inlet Reynolds number, $\langle Re \rangle$, which is the pump discharge per unit channel width divided by the kinematic viscosity of the liquid. This Reynolds number is used in the majority of experimental studies of falling films. For fixed liquids, it is the only free parameter.

On the other hand, when we consider individual localized structures, it is convenient to introduce another Reynolds number, the Reynolds number of the soliton substrate, Re : as we first pointed out in Sec. II of Part I and Sec. III of Part I, far from its hump a solitary wave decays rapidly to a flat flow, the so-called “substrate” or “sublayer”. Re is based on the substrate’s thickness. Note that the substrate Reynolds number has been used in the past by several authors, see for example [11].

Here, we are interested in the connection between the two Reynolds numbers, $\langle Re \rangle$ and Re . In the case of wave-less flow, the two numbers are the same, $\langle Re \rangle = Re$, but for a wavy regime the situation is quite different. In this case, the connection between the two Reynolds numbers depends on the number of 2D or 3D solitary waves per unit area in the (x, z) -plane or, equivalently, the “density” of these “quasi-particles”.

Let us first define the volumes of 2D and 3D solitary waves, respectively,

$$J_{2D} = \int_{-\infty}^{\infty} (h - 1) dx \quad (1)$$

and

$$J_{3D} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (h - 1) dx dz, \quad (2)$$

where (2) was first introduced in Eq. (19) of Part II but is rewritten here for clarity. As we pointed out in Sec. II D of Part II, even though at large δ the amplitude of 3D solitons

tends to a constant, their volume J_{3D} continues to increase. For 2D solitons, we have similar behaviour, i.e. for large δ their amplitude saturates but their volume continues to increase with δ due to the widening of their humps. In fact, for large δ , the volume J_{2D} of 2D solitons is a linear function of δ , much like the volume of 3D solitons, and can be well approximated by the relationship

$$J_{2D} = 208\delta + 0.535$$

For completeness, we also give the volume of 3D solitons for large δ from Sec. II D in Part II:

$$J_{3D} = 2034\delta - 81.54$$

Let us assume that we have a wave-less substrate of unitary thickness with Reynolds number Re (the corresponding dimensional thickness is \tilde{h}_0 — see Sec. II of Part I). Let us also assume that a group of 3D localized coherent structures, Λ -waves, with average distances between them L_x and L_z , in the streamwise and spanwise directions respectively, slides with a speed c along this substrate. We also define the wave numbers, $\alpha = 2\pi/L_x$ and $\beta = 2\pi/L_z$ in the x - and z -directions respectively. Let us single out one “quasi-particle” and integrate the mass balance equation (5c) in Part I in the frame moving with speed c along a period in the z -direction and use the fact that the z -component of the flow rate, p , is zero far from the hump,

$$\frac{\partial}{\partial x} \left(Q - \frac{c}{L_z} \int_{-L_z/2}^{+L_z/2} h dz \right) = 0 \quad (3)$$

where $Q = Q(x)$ is averaged with respect to the z flow rate,

$$Q = \frac{1}{L_z} \int_{-L_z/2}^{+L_z/2} q dz$$

Integrating (3) once with respect to x gives

$$Q - \frac{c}{L_z} \int_{-L_z/2}^{+L_z/2} (h - 1) dz = 1 \quad (4)$$

where the integration constant was determined from the condition that the soliton decays rapidly far from its hump to plane-parallel flow $h = 1, q = 1$ (Eq. (7), Part I). Further, we integrate (4) once with respect to x :

$$\langle Q \rangle = 1 + \frac{c}{L_x L_z} \int_{-L_x/2}^{+L_x/2} \int_{-L_z/2}^{+L_z/2} (h - 1) dx dz \quad (5)$$

where

$$\langle Q \rangle = \frac{1}{L_x} \int_{-L_x/2}^{+L_x/2} Q dx$$

is an averaged flow rate along the x -direction. If L_x and L_z are sufficiently large, the double integral on the righthand side of (5) can be approximated by J_{3D} in (2), giving the final relationship

$$\langle Q \rangle \sim 1 + \frac{c J_{3D}}{L_x L_z} = 1 + \frac{\alpha \beta c J_{3D}}{4\pi^2}. \quad (6)$$

The first term is the contribution from the slowly moving thin substrate; the second term is the contribution from the rapidly moving large hump. It depends on the “density” of pulses $\alpha\beta$, velocity of the wave c and its volume J_{3D} . Mass conservation between the flat inlet region and the region of well-developed waves further downstream implies that $\langle Q \rangle$ is equal to the dimensionless inlet flow rate. Hence, the inlet Reynolds number $\langle Re \rangle$, based on inlet flow rate, and the sublayer Re based on sublayer flow rate (whose dimensionless value = 1), are connected by the relation

$$\langle Re \rangle = \langle Q \rangle Re. \quad (7)$$

We can also derive for the 2D wave regime an expression similar to (5):

$$\langle Q \rangle = 1 + \frac{c J_{2D}}{L_x} = 1 + \frac{\alpha c J_{2D}}{2\pi} \quad (8)$$

From Eqs. (5) and (6), in the 3D-soliton regime, if either volume $J_{3D} \rightarrow 0$ or “density” $\alpha\beta \rightarrow 0$ (equivalently “volume” $J_{2D} \rightarrow 0$ or “density” $\alpha \rightarrow 0$ in the 2D-soliton regime), then $\langle Q \rangle \rightarrow 1$ and $\langle Re \rangle \rightarrow Re$, as would be the case for a single soliton in an infinite domain. However, usually the contribution of the second term in (5) is much larger than the first term and $\langle Re \rangle$ is much larger than Re .

The boundary of the 2D–3D transition in experiments is about $\langle Re \rangle \approx 40 - 70$ for water (see Fig. 1, Part I). We should compare these values with our theoretical critical $\delta_* = 0.048$ or $Re_* = 5.5$ for water (Sec. IV, Part I), which correspond to instability of 2D solitons. In order to do this, we should recalculate Re as $\langle Re \rangle$ and apply relations (5)–(7).

However, this can only be done qualitatively because the “density” of the soliton gas and hence $\langle Q \rangle$ is unknown. Nevertheless, we can use the results of previous experiments to estimate the second term in (5) and hence evaluate $\langle Q \rangle$. According to Chu and Dukler [12], the contribution of the second term can be ten to twenty times larger than that of the first one, so the majority of the inlet flow rate is carried by solitons. For our range of Reynolds numbers, we take the second term to be ten times the first one (see Fig. 12 of Chu and Dukler’s paper). This gives us the rough estimate

$$\langle Re \rangle = 10Re. \quad (9)$$

Hence, our transition Reynolds number $Re = 5.5$ is equivalent to $\langle Re \rangle = 55$, which is in good agreement with the experimental value $\langle Re^{(2)} \rangle \approx 40\text{--}70$ where the 2D–3D transition takes place (Fig.1, Part I).

An important related question is which type of solitons, 3D or 2D, carries more liquid. This question, in particular, has important consequences for heat/mass transfer. As we demonstrated in Figs. 11 and 12 of Part II, both the amplitude and speed of a 3D soliton are smaller than those of a 2D soliton of the same δ . On the other hand, as δ increases, a 3D soliton becomes bulkier, and 3D solitons can potentially carry more liquid than 2D solitons for the same δ . By equating (8) to (6), we obtain the limiting value β_* ,

$$\beta_* = 2\pi \frac{\alpha_{2D}}{\alpha_{3D}} \frac{c_{2D}}{c_{3D}} \frac{J_{2D}}{J_{3D}}. \quad (10)$$

If $\beta > \beta_*$, 3D solitons carry more liquid than 2D ones; if $\beta < \beta_*$, 2D solitons carry more liquid than 3D ones. Analysis of experimental data shows that the average separation distance L_x for 3D waves is much smaller than for 2D, in other words, 3D waves are “better packed” in the same domain than 2D waves. As a result α_{2D}/α_{3D} is small which together with the fact that J_{2D}/J_{3D} is small (Eq. (1) implies from (10) that β_* is small as well). Hence, provided that the average separation distance L_z of 3D waves is not very large, and the experiments show that it is not, 3D waves carry more liquid than 2D ones. Hence, both cases are possible, depending on experimental conditions.

3 Experiment

Experimental setup.

We are going to investigate the range of Reynolds numbers $\langle Re \rangle$ from 5 to 100. For such Reynolds numbers, previous works have analyzed the flow of liquid films falling down vertical tubes with diameters from 2 to 4 cm and with water as test liquid. However, the typical soliton size in our setup is 3 to 7 cm in the spanwise direction, so such tube diameters are too small. We employ an experimental apparatus with a planar vertical channel, 25 cm long and 15 cm wide and made of a special mirrored glass, see Fig. 1.

A digital pump is located in the lower part of the setup since the flow is sensitive to external vibrations. Liquid is pumped through a rubber pipe to a supply tank of size 25 cm \times 15 cm \times 10 cm. In the upper part of the tank, there is a feeding device of overflow type and the liquid flows out of it through a special polymer net onto the vertical test surface. The net serves to decrease the tangential velocity of the liquid. From the test section, the liquid flows to the discharge tank. The setup forms a closed loop, such that the liquid returns from the discharge tank to the pump.

Distilled water is used as working fluid. The temperature is kept at $15 \pm 0.5^\circ\text{C}$, so that the viscosity is $\nu = 1.14 \times 10^{-6} \text{ m}^2/\text{s}$, the ratio of surface tension to density is $\sigma/\rho = 74 \times 10^{-6} \text{ m}^3/\text{s}^2$ and the Kapitsa number is $\gamma = 2900$. The Reynolds number at the inlet, $\langle Re \rangle$, is measured as the discharge flow per unit width divided by the viscosity ν . The discharge volumetric flow rate of the pump is varied from 7 to 150 cm^3/s and hence the corresponding range of Reynolds numbers is $4.5 < \langle Re \rangle < 100$.

It is well known that the main difficulty with an overflow feed in falling film experiments is the formation of a uniform flow at the inlet and further downstream. With water as working liquid, the film can be very thin. As a result, a small disturbance of the overflow at the inlet leads to the liquid breaking up into drops and rivulets. To remedy this effect, we take the following measures: a) the upper edge of the channel is aligned strictly perpendicular to the gravity field; b) the channel has a small width, 15 cm, as mentioned earlier; c) a polymer net is installed at the inlet to decrease the tangential component of velocity; d) in the inlet region, we engrave ten small parallel ditches streamwise, each

15 mm long and of triangular cross-section. These measures significantly minimize the formation of drops and rivulets. Still, at small Reynolds numbers, $\langle Re \rangle < 3 - 4$, we are not able to create a plane-parallel substrate. Finally, polished steel side walls are used to minimize the side-wall effect associated with flows in rectangular channels.

Methods to measure wave characteristics.

To measure 3D wave profiles and their characteristics, we use the fluorescent imaging method by Liu and Gollub [7]. The method was also successfully used by Vlachogiannis and Bontozoglou [8]. A fluorescent dye with a low concentration (about 100 - 150 ppm), with chemical formula $C_{20}H_{10}Na_2O_5$, is added to the fluid. Such a small concentration does not affect the liquid physical properties [7]. Eight ultraviolet lamps are arranged over the channel surface to illuminate it from above and a chamber is arranged on the opposite side of the channel. The absolute accuracy of the method is 8 to 10 μm , hence, the largest possible relative error varies from 6 to 8 %.

The local film thickness can be obtained from the signal by careful calibration. To perform the calibration of film thickness versus light intensity, a part of the layer near the distributor is used. It remains flat up to the inception point which varied from 7 to 25 cm. In the measurements, we avoid a small portion of the channel just after the inlet to let the flow establish a steady semi-parabolic profile (around ten to twenty film thicknesses). The flat portion is used for calibration along with the known discharge of the pump. The obtained dependence is shown in Table 1. Between these points, a linear interpolation is used,

$$I(x, z, t) = K * I_0(x, z)h(x, z, t) + I_1(x, z)$$

where K is constant, and $I_0(x, z)$ and $I_1(x, z)$ depend on the location because of the non-uniformity of the ultraviolet light field.

The digital analogue of the measured physical signal of the surface is recorded. Photographs are taken with a high-resolution Panasonic NV-GS75 digital camera. The data are recorded on a digital carrier and further processed using a computer with a spatial resolution of 720 x 540. In order to measure the wave speed, a series of frames of the wave are captured at $1/30 - th$ of a second apart.

Typical examples of shadow images of waves, their fluorescence images and cross-sections for forced 2D periodic waves with $\langle Re \rangle = 30$ and frequency $f = 15.0$ Hz and for a 3D signal with $Re = 6.8$ are shown in Fig. 4.

To measure the wave's amplitude by fluorescence imaging, we use a simple contact method in which a steel needle is positioned in the lower part of the channel. The needle could move back and forth by means of an adjusting micrometre screw. The flat surface of the substrate is used as a reference point. Both electrical and visual methods are employed. In the first case, the contact is registered by closing the electrical circuit between the needle and contacting liquid. A very small amount of sodium chloride which does not affect the hydrodynamical properties of the working liquid is added to it to make it conductive. The circuit includes a 4-volt battery and an ampere-meter. To measure the maximum amplitude of the wave, we need only one contact with the surface after which the experiment is stopped. In the visual method, the circuit is switched off and on by the top of the needle where a small coloured particle is positioned. When the surface is touched by the needle, the particle is taken by the flow. A microscope is employed to attach the particle and to decide if the particle is taken by the flow or not.

Methods to investigate 2D-3D transition and 3D solitary waves.

In order to form 2D solitary waves, localized periodic pulsations in flow rate are superimposed on the main flow [9, 10]. The resultant “artificial” waves suppress the natural perturbations and a rather regular train of 2D solitary waves travelling downstream is generated. In order to induce a 3D instability and the process of disintegration of 2D solitary waves into 3D localized coherent structures, we impose artificial perturbations onto the 2D solitons. For this purpose special needles are placed in the transverse direction at the inlet, with a separation distance \tilde{l} from each other, as was done in [3]. We perform experiments with \tilde{l} from 10 mm to 35 mm in steps of 2.5 mm.

Let us now discuss how to create 3D solitary waves. Previous falling film experiments show that 3D solitons can be seen after a long and complex downstream evolution [13, 14] (see also Fig. 1, Part I). The key idea of our work is to bypass all intermediate stages of evolution and deposit a single drop with a fixed mass onto the smooth film surface.

The advantage of our method is that we deposit a drop with a mass close to that of the theoretical 3D solitary wave. We refer to this method as the “raindrop” method. It was prompted literally by the rain on a car windscreen during a traffic jam. This method provides a simple, easy to implement and efficient way to successfully create individual 3D solitons and to measure their parameters as a function of the substrate Reynolds number.

The depositing of drops is done with a special small Finn pipette with adjustable volume. We have found this device to be the most appropriate for our purposes. To cover the range of soliton volumes obtained in Part II, reproduced here for the ease of presentation in Table 2, we employ three Finn pipettes with the following drop volumes: $0.5 - 10 \mu\text{L}$ in increments of $0.1 \mu\text{L}$; $5 - 40 \mu\text{L}$ in increments of $0.5 \mu\text{L}$ and $40 - 200 \mu\text{L}$ in increments of $1 \mu\text{L}$. The smallest diameter of the liquid drop is 2 mm and the largest is 7 mm. The calculated dependence of soliton mass on Reynolds number in Table 2 is used to estimate the mass of the liquid drop. More specifically, the mass of the liquid drop is taken as 1.1 to 1.2 times that of the equilibrium mass. Having a liquid drop mass close to the steady 3D soliton mass results in rapid formation of a 3D soliton.

In the experiment, the pipette is attached to the upper part of the channel such that its distance and angle to the surface can be changed. The most painstaking procedure is to adjust the pipette in such a way that the moment the required drop size is achieved the drop touches the film surface and is subsequently involved in film movement. A schematic picture is given in Fig. 7. We have two degrees of freedom to approach the film surface, distance and angle to the surface, which are adjusted independently for all the flow rates used. During adjustment, the image of the drop is magnified by a strong lens located near the pipette. The distance and angle to the surface are changed by employing adjusting micrometre screws. Finally, a table of angles and distances as a function of flow rate is obtained. After the drop touches the surface, it is subsequently involved in film movement and is rapidly transformed into a stationary localized wave.

The important condition in this method is to have a flat film surface for the deposited drop. Recall that the film remains flat from the inlet up to the inception point. The

length of this region is varied from 7 to 25 cm. The drop is deposited onto this part of the flow.

We found that it takes from 5 to 10 cm for the Λ -wave to reach a steady state — the unstable ripples at $\langle Re \rangle < 15$ are too weak to affect the formation of the Λ -wave in the upper part of the channel. In this region of Reynolds numbers, the upper part of the flow can be assumed to be practically flat. At $\langle Re \rangle \simeq 20$, room disturbances have a sufficiently large growth rate to turn into natural waves; although they continuously disturb the configuration of a Λ -wave sliding on a flat substrate, they are not able to destroy it: the Λ -structure also grows with increasing Reynolds number and is sufficiently robust to withstand the natural waves.

The details regarding the apparatus and the experimental techniques can be found elsewhere [15].

Numerical experiment.

To complete our investigation, we perform numerical modelling of the non-stationary process of Λ -soliton creation from a drop as done in the physical experiments. As in [1, 2], we use the following Kapitsa-Shkadov system of three nonlinear PDEs:

$$\begin{aligned} \frac{\partial q}{\partial t} + \frac{6}{5} \frac{\partial}{\partial x} \frac{q^2}{h} + \frac{6}{5} \frac{\partial}{\partial z} \frac{qp}{h} &= \frac{1}{5\delta} \left(h \frac{\partial}{\partial x} \nabla^2 h + h - \frac{q}{h^2} \right), \\ \frac{\partial p}{\partial t} + \frac{6}{5} \frac{\partial}{\partial x} \frac{qp}{h} + \frac{6}{5} \frac{\partial}{\partial z} \frac{p^2}{h} &= \frac{1}{5\delta} \left(h \frac{\partial}{\partial z} \nabla^2 h - \frac{p}{h^2} \right), \\ \frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} + \frac{\partial p}{\partial z} &= 0, \end{aligned} \tag{11}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial z^2$ is the Laplace operator on the (x, z) plane. All the values are dimensionless and incorporate the liquid density ρ , thickness h_0 and averaged velocity $u_0 = (gh_0^2)/(3\nu)$ on the flat substrate. The further transformation,

$$x \rightarrow \kappa x, \quad z \rightarrow \kappa z, \quad t \rightarrow \kappa t, \quad v \rightarrow v/\kappa, \quad \kappa = \frac{\gamma^{1/3}}{3^{2/9} Re^{2/9}}$$

allows to absorb two parameters, the substrate Reynolds number Re and Kapitsa number $\gamma = \sigma \rho^{-1} \nu^{-4/3} g^{1/3}$, into a single parameter,

$$\delta = \frac{Re^{11/9}}{3^{7/9} 5 \gamma^{1/3}}.$$

Here, σ is the surface tension, ρ and ν are the liquid density and viscosity, and g is the acceleration of gravity.

The numerical method of ^[1] is used, which is an extension to 3D of the 2D scheme developed in the studies of ^[16]. For a localized 3D signal, the calculations are performed in a frame moving with the velocity of the signal. The computational domain is $2L_x$ x $2L_z$. A sufficiently large size for the domain allows the final Λ -wave solution to be free from “side-wall” effects, or, equivalently, the localized wave has sufficient space to decay almost to zero at the boundaries of the domain. We impose boundary conditions at both ends of the domain,

$$\begin{aligned}
x = -L_x : \quad h &= q = 1, \\
x = +L_x : \quad h &= q = 1, \quad p = 0, \\
z = \pm L_z : h(x, -L_z, t) &= h(x, L_z, t), \\
q(x, -L_z, t) &= q(x, L_z, t), \\
p(x, -L_z, t) &= p(x, L_z, t),
\end{aligned} \tag{12}$$

The initial conditions are taken in the form,

$$t = 0 : \quad h = 1 + Ae^{-b(x^2+z^2)}, \quad q = 1, \quad p = 0 \tag{13}$$

which simulates a drop on the film surface with parameters A and b describing the amplitude and spreading of the drop, respectively.

4 Results

Visual observations of natural waves and their transitions.

First, we consider experiments with waves originated from natural room disturbances. A visual examination of the surface shows that it is absolutely flat at $\langle Re \rangle < 4 - 5$, confirming previous observations, $\langle Re^{(1)} \rangle \approx 4 - 5$. Note that even at $\langle Re \rangle = 7$, the manifestation of waves is very weak, see Fig. 3(a). For larger Reynolds numbers, the surface is covered with waves. At any $\langle Re \rangle$, experiments show the presence of a flat region near the inlet up to the inception point. At $\langle Re \rangle \geq 7$, the points of wave inception

are located on a line at the following distance from the inlet: $\langle Re \rangle = 7 - 25$ cm; $10 - 15$ cm; $15 - 10$ cm; $20 - 7$ cm. This distance is a little larger than cited in other works (see [13]) and [16]). We attribute the difference to the small width of the channel and wall effects.

For Reynolds numbers $\langle Re \rangle$ from 7 to 15 – 20, the waves are two-dimensional and periodic. For Reynolds numbers from 15 – 20 up to 40 – 50, the waves maintain the basic structure of 2D solitons but are disturbed with some 3D modulation, either stationary or pulsating. Localized 3D waves can be seen in this range of Reynolds numbers but they have small amplitudes and, what is more important, they have a short lifetime and disappear rapidly downstream.

The 2D–3D transition occurs approximately at $\langle Re^{(2)} \rangle \approx 40 - 50$, see Fig. 3(b). The 3D modulation now is not stationary any more, but grows downstream with eventual destruction of the 2D wave and appearance of steady 3D solitons. We note that for $\langle Re \rangle > \langle Re^{(2)} \rangle$, the coalescence of neighbouring 2D waves increases the 3D modulation and accelerates the destruction process. A typical well-developed 3D regime is shown in Fig. 3(c). The following interesting visual observation is made regarding the ST: if we look at a fixed point on the channel, thus taking the Eulerian viewpoint, the evolution appears to be purely chaotic. But if we follow the waves with an appropriate speed, thus taking the Lagrangian viewpoint, we can decipher interacting deterministic localized structures, which look like “quasi-particles”. In other words, the ST can be considered to be a chaotic interaction of quasi-particles which are strongly internally coupled and continuously interact with their neighbours. An example of the chaotic wave motion in this region is shown in Fig. 6 for $\langle Re \rangle = 60$. Several typical patterns can be identified: 1 — 2D soliton with 3D modulation prior to being destroyed downstream; 2 — Well-developed 3D solitons interacting through their overlapping tails; 3 — Complex coalescence of several 3D solitons.

The 3D localized coherent structures slide “rapidly” on a “slowly” moving thin sub-layer with Reynolds number Re . In the 3D wave regime, most of the liquid is concentrated in the 3D structures and only a small fraction of it is carried by the thin substrate. As

we pointed out in the previous section, for our range of Reynolds numbers we take this fraction as ~ 10 . As a consequence, the above range of inlet Reynolds numbers for 3D wave regimes translates into $4 < Re < 40$.

Forced waves and comparison with our theory.

The rather irregular picture of natural waves can be improved by imposing artificial periodic pulsations in flow rate on the main flow. Further downstream, the pulsations are convected into regular 2D periodic waves, at low pulsation frequency the periodic waves turn into 2D solitary waves. The initially 2D waves are disturbed in the transverse direction by a system of needles placed with a separation \tilde{l} and the regular structure of waves is immediately violated. At some distance from the inlet, 3D modulations appear to be superimposed on our 2D solitons. These modulations grow downstream and eventually destroy the 2D solitons, and 3D localized structures appear at some distance \tilde{L} from the inlet. A typical snapshot of such an evolution is shown in Fig. 4, $\langle Re \rangle = 10$, $\tilde{l} = 25$ mm.

We perform experiments with different \tilde{l} , from 10 mm to 35 mm, in steps of 2.5 mm. For sufficiently short separation between the needles, more specifically, $\tilde{l} \leq 10$ mm, there is no sustained reaction and the 3D perturbations decay downstream. At larger \tilde{l} , wave disintegration into localized 3D coherent structures at a distance \tilde{L} is observed. From one experimental run to another, this distance varies with an amplitude of approximately 0.5 cm. Our data are averaged over 20 to 40 points to obtain \tilde{L} . The results of our observations for \tilde{L} are presented as a function of the distance \tilde{l} for $\langle Re \rangle = 10$ in Fig. 5. The dependence has a pronounced minimum at $\tilde{l} = 2$ cm which is in good agreement with our theoretical prediction ^[1] of the most dangerous wavelength, $\tilde{l}_m = 1.42Re^{1/9} = 1.83$ cm, and with experimental observations ^[3]. Moreover, the downstream distance for complete disintegration, \tilde{L} , is about 10 to 15 cm, which also fits the theoretical prediction 8 – 15 cm (Part I Fig. 10).

In the raindrop method, a 3D coherent structure is formed by depositing a liquid drop onto the film surface. The drop is subsequently involved in film movement. We emphasize that the drop's evolution results in a stationary 3D solitary wave only if the drop's mass is close enough to the equilibrium one. In this case, the drop is rapidly transformed

into a stationary localized wave. It takes about 5 to 10 cm from the location where the drop touches the film surface for a stationary structure to form. For a larger mass, approximately as much as twice the equilibrium mass, a two-hump 3D solitary wave is formed. This wave is unstable and decomposes further downstream into two one-hump 3D solitary waves. If the mass of the deposited drop is much smaller than the equilibrium mass, that is approximately one half, then it never evolves into a stationary wave but expands in a 3D wave packet.

A comparison of experimental and theoretical cross sections of Λ -solitons is given in Fig. 8. There is a rather good correspondence between the wave profiles, except for the capillary ripple region in front of the wave — theoretical oscillations are more pronounced. This can be explained by insufficient accuracy in our measurement in the region of capillary ripples, $8 - 10 \mu m$. Also, the theoretical trough behind the hump is deeper than the experimental one, the difference becoming larger at large δ .

The Λ -wave profiles and other wave characteristics can also be found in experimental works [4, 5]. In [5], parameters for two solitary waves are presented for $\gamma = 404$: $Re = 2.5$, $\delta = 0.035$, and $Re = 3.9$, $\delta = 0.061$. According to our theory in Part II, at $\delta < 0.054$ the Λ -soliton is unstable against the radiation mode, in other words at small δ , localized disturbances of the plane-parallel part of the Λ -wave destroy the hump. Indeed, the first experimental wave looks unsteady and shows some indication of further disintegration. In Fig. 9, we compare our theoretical (a) and experimental (b) wave profiles and their cross-sections for the second case, $Re = 3.9$, $\delta = 0.061$, $\gamma = 404$. The experimental and theoretical velocities are 209 and 213 mm/s, respectively. For this δ , the wave is stable and we can find good quantitative agreement between our theory and experiment. Our theory [17] also predicts the existence of two-hump 3D solitons and their characteristics. Such solitons were found experimentally in [4]. The comparison of theoretical and experimental data in Fig. 10 gives reasonably good correspondence.

Experimental data for the wave speed is given in Table 3. For the first two Reynolds numbers, we are not able to create a 3D soliton because of its instability. Instead of a nonlinearly coupled localized structure, we observe a spot of expanding chaotic waves

confirming that at $\delta < 0.054$ ($Re < 6$) 3D solitons are unstable, in agreement with our theoretical predictions. We are not able to sustain a stationary 3D solitary wave at $\delta \leq 0.054$ because of a strong primary instability of the flat region. However, at larger δ , the radiation is convected away from the 3D soliton and it is stable and robust. The dimensional values from Table 3 are plotted in Fig. 11 along with the theoretical ones. The agreement is really good especially taking into account the simple way our experiment is performed.

Fig. 12 compares the experimentally observed wave amplitudes as a function of Reynolds number with the theoretical ones. Again, the agreement is rather good.

Numerical experiment — comparison with physical results.

Typical numerical results of the time-dependent evolution for $\delta = 0.0908$ (for water $Re = 9.28$) are shown in Fig. 13. A localized ‘drop’ with the initial form described by (13) and with a mass 20% larger than the equilibrium 3D soliton mass is placed on the flat film substrate at $t = 0$. At $t = 0.05$ the drop has a tendency to split into two humps; however, provided this event is avoided, instead of disintegration one nonlinear hump of the soliton is formed. (The tendency to split into two humps cannot be arrested if we take a drop with an initial mass at least twice the equilibrium 3D soliton mass.) At time $t = 1$ the signal acquires capillary ripples (this moment is skipped in Fig. 13), and at $t = 5$ the trough behind the head develops. At $t = 12 - 20$ the 3D soliton is practically formed. One can clearly see at this stage the excess mass draining slowly to the back, leaving the equilibrium 3D soliton behind it. The final structure is characterized by capillary oscillations/ripples at its front along with moustache-like or leg-like structures at the back and a long hollow trough between the legs. The ditch is caused by a Bernoulli depression in the moustaches and in fact the numerical experiments performed here indicate that it develops simultaneously with the development of the moustaches. A completely stationary Λ -wave running at speed $c \simeq 4.6$ is formed at $t = 20$.

The results of these calculations are compared with our experiments in Fig. 14, where the wave amplitude h_{max} is presented as a function of the distance downstream. The

theoretical time is recalculated as the distance downstream using the signal velocity and physical properties of water. The dimensionless time of 12 to 20 corresponds to a dimensional distance of 6 to 10 cm which is in a good agreement with the physical experiments.

The calculations also confirm other experimental results: If the mass of the “drop” is at least twice as large as the equilibrium 3D soliton’s mass, the drop disintegrates into two or several solitons or into a two-hump 3D soliton. If the drop’s mass is smaller than the equilibrium 3D soliton’s mass, the structure sucks up liquid from the substrate as it evolves and it takes two to three times longer to reach the final Λ -wave’s steady state. Moreover, starting with a small initial mass, the evolution never results in a stationary 3D wave but in an expanding and growing wave packet.

The evolution of different shapes of initial signal was also investigated. The constants A and b in (13) are varied in such a way that the volume of the initial signal remains the same. In all cases, the main stages of the evolution appear to be qualitatively the same. Interestingly, the time to reach a 3D stationary structure appears to be almost independent of the form of the initial signal. Furthermore, changing the initial mass of the drop to 1.5 times that of the equilibrium 3D solitary wave also gives qualitatively the same stages of evolution and finally results in a stationary 3D pulse with nearly the same time of saturation $t \simeq 10$ to 20.

It is now well established that the interaction of 2D solitons with smaller objects in front of them consists of a coalescence process in which the solitons absorb the mass of the smaller objects. This results in an increase in the speed and amplitude of the solitons by an amount proportional to the mass of the smaller objects. This coalescence process also takes place between 2D solitons of different amplitudes and speeds: the faster soliton catches up with the smaller one and absorbs its mass. The coalescence event between 2D solitons has been observed experimentally ([10]) and has been described theoretically [18].

Fig. 15 presents some computational results of such an interaction, but for 3D solitons. At $t = 0$ we place in front of a stationary 3D soliton a localized signal with a mass of

$\frac{1}{2}$ the mass of the soliton. At $t = 5$, the soliton absorbs the mass of the localized signal and retains it until $t = 25$, i.e. for a time interval of $\Delta t = 20$. Recalculation of this dimensionless time into a dimensional distance gives roughly 10 cm. The coalescence event causes an instantaneous acceleration and increase in speed/amplitude proportional to the absorbed mass. The 3D soliton is now an “excited” soliton, i.e. a soliton of larger amplitude than that corresponding to the given δ . For $t > 25$, drainage of the excited soliton begins and the excess mass leaves the wave. It is expelled from the structure through its “moustaches” and eventually takes the form of two small 3D solitons ($t = 41$). These solitons suck up liquid from the substrate and eventually grow into equilibrium 3D solitons. This is the key mechanism for generating the staggered “checkerboard” pattern seen in many 3D falling film experiments. The beginning of this process is seen at $t = 45$ and 49.

5 Conclusions

Through simple experiments, we have examined the instabilities and transitions leading to the regime of ST. We have shown that if 2D solitary pulses are unstable to transverse perturbations, their disintegration results in the formation of 3D Λ -solitons. For water, at sufficiently large Reynolds numbers the most dangerous length is 2 cm; this result is close to that obtained theoretically ^[1].

We have developed a new experimental technique, the “rain-drop” method, to investigate Λ -solitons and their characteristics. Wave shapes, amplitudes and velocities have been obtained for a wide range of Reynolds number. Numerical experiments modelling “rain-drop” formation have also been performed. We have established good agreement between the theoretical and experimental characteristics and parameters of Λ -solitons.

An important next step in our investigation is the development of a statistical theory of the ST. We also plan an experiment and a direct numerical simulation of the 3D wave regime in a vertically falling film to model in detail the downstream evolution of natural waves, their interaction and statistics. In order to extract the deterministic features of the 3D wave regime, the methods of “spatial averaging” and “proper orthogonal”

decomposition used successfully in ordinary turbulence ^[19] might also prove very useful in the falling film problem. We hope to examine these and related problems in future studies.

Table 1 Calibration of thickness vs. intensity for the flat film.

$h, \mu m$	I
0	0
141	24.8
146	25.5
152	26.3
155	26.7
163	27.8
168	28.5
173	29.2
183	30.6
191	31.6
311	47.1
503	69.5

Table 2 Stretching coefficients κ , Reynolds number of substrate and volume of equilibrium 3D-“drops”, dimensionless and dimensional, as a function of δ for water, $\gamma = 2900$.

δ	κ	Re	J_{3D}	$\tilde{J}_{3D}, \mu L$
0.005	11.48	0.861	3.054	0.1378
0.010	10.12	1.518	4.748	0.2935
0.015	9.403	2.116	5.775	0.429
0.020	8.924	2.677	7.327	0.621
0.025	8.569	3.213	8.739	0.819
0.030	8.289	0.373	10.483	1.068
0.035	8.060	4.232	12.720	1.389
0.040	7.867	4.720	15.491	1.798
0.045	7.700	5.197	19.374	2.373
0.050	7.554	5.666	24.518	3.150
0.55	7.424	6.125	31.920	4.283
0.060	7.308	6.577	40.862	5.704
0.070	7.106	7.462	63.692	9.536
0.090	6.788	9.165	101.52	17.039
0.150	6.187	13.92	223.56	47.330
0.200	5.871	17.61	325.26	78.481
0.500	4.970	37.28	935.46	343.31

Table 3 Pipette drop volume J , Reynolds number Re , wave velocity c , and maximum thickness of wave h_{max} , as a function of supply tank volumetric discharge rate, Q (water).

$Q, cm^3/s$	Re	$J, \mu L$	$c, cm/s$	h_{max}, mm
7.50	4.38	2.00	-	—
9.00	5.26	3.00	-	—
10.5	6.72	5.00	22.5	0.20
12.0	7.02	9.00	24.5	0.28
13.5	7.89	15.0	27.7	0.32
15.0	8.77	20.0	31.1	0.35
16.5	9.65	25.0	33.9	0.41
19.5	11.40	40.0	37.5	0.39
21.0	12.28	45.0	39.3	0.40
22.5	13.16	60.0	42.6	0.43
24.0	14.03	60.0	43.2	0.47
24.0	14.03	70.0	42.1	0.48
27.0	14.91	75.0	41.1	0.51
27.0	15.79	80.0	46.0	0.47
28.5	16.67	90.0	47.8	0.48
30.0	17.54	100.0	48.9	0.50

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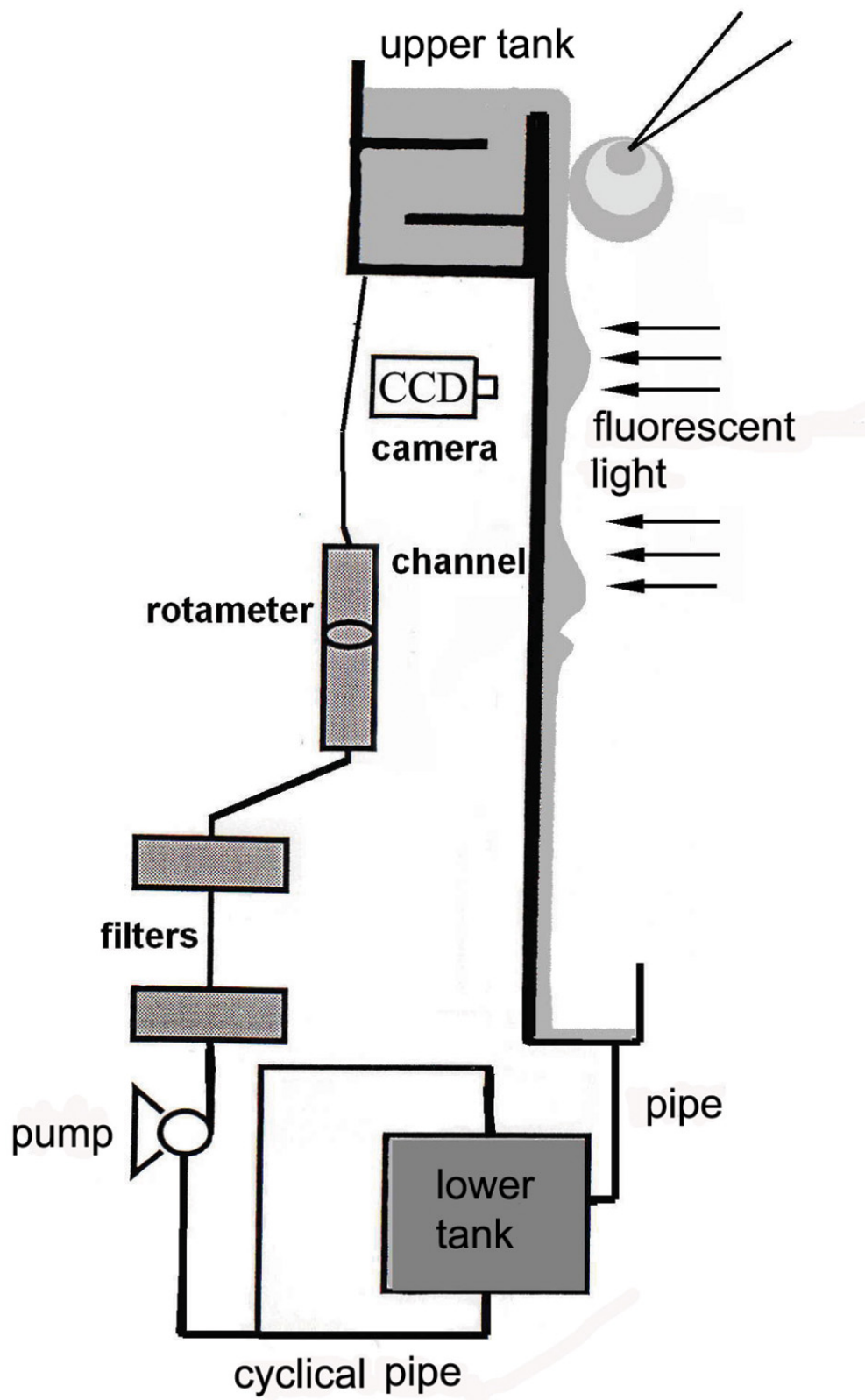


Figure 1: Experimental setup.

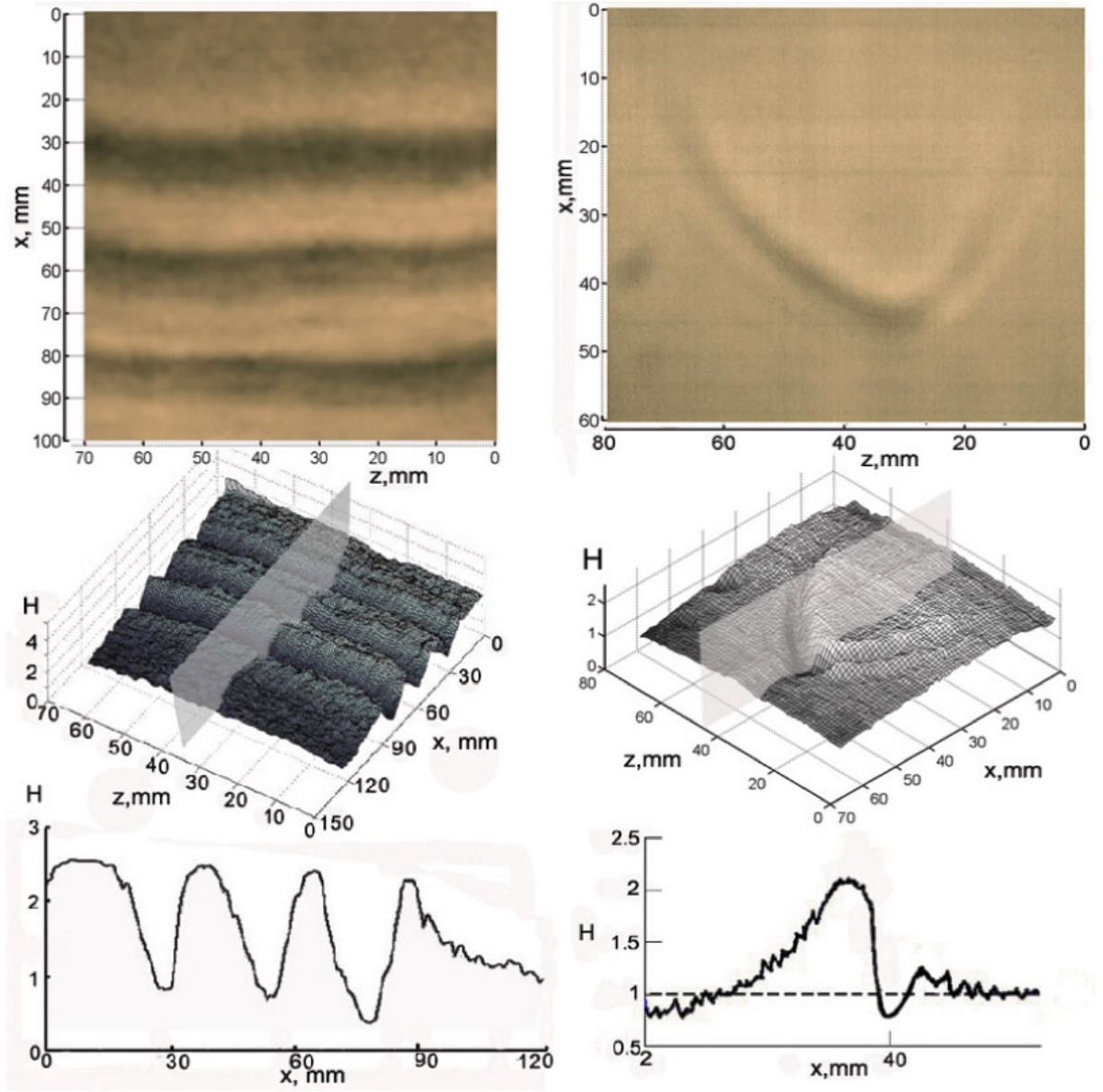


Figure 2: Examples of shadow images of waves, fluorescence images and cross sections: (a) Forced 2D periodic waves for $Re = 30$ and frequency $f = 15.0$ Hz. (b) 3D-soliton for $Re = 7$.

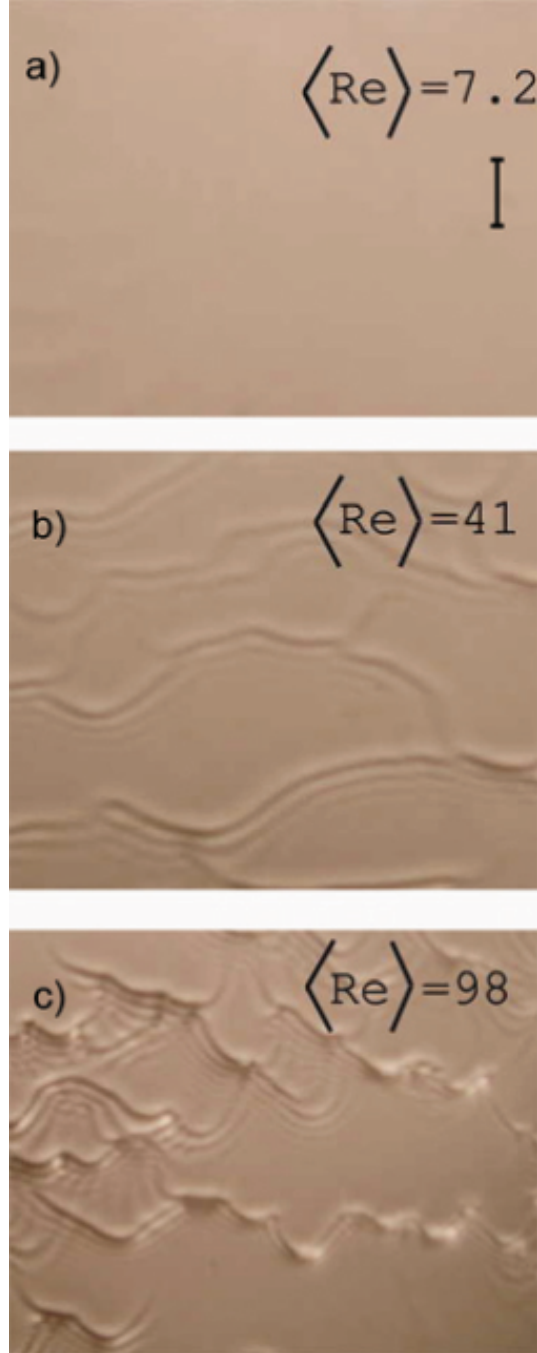


Figure 3: Shadow images of waves generated by room disturbances. The bar is 2 cm long. (a) $\langle Re \rangle = 7$, the flow is practically flat, (b) $\langle Re \rangle = 41$, 2D-3D transition and (c) $\langle Re \rangle = 98$.

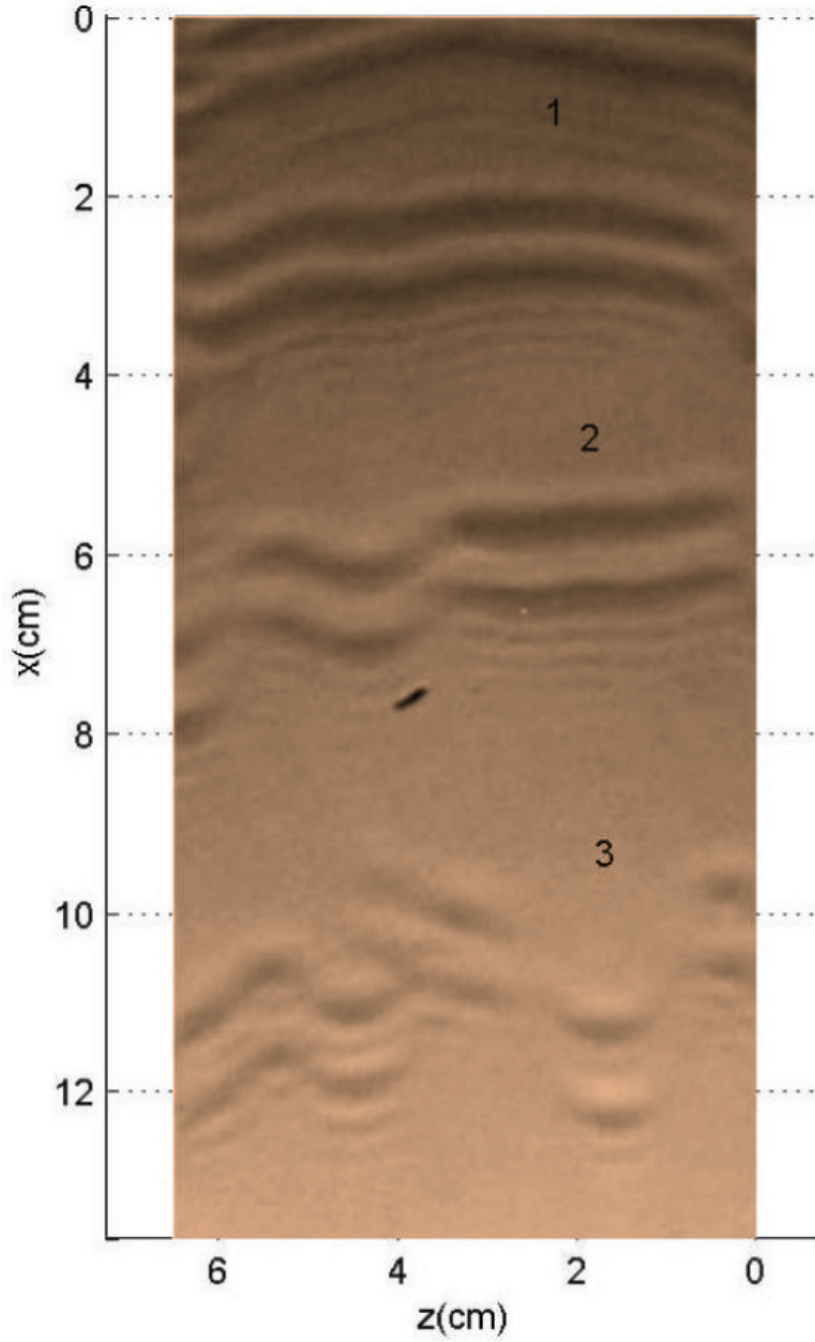


Figure 4: Shadow image of periodically excited waves for $Re = 10$. The waves are disturbed in the spanwise direction with needles at the inlet with separation $\tilde{l} = 25$ mm. At point 1, the wave is practically two-dimensional; at point 2 at a distance of 6 cm from the inlet, one can clearly see the 3D modulation, at point 3 at a distance $\tilde{L} = 11$ cm from the inlet, the original waves are completely destroyed.

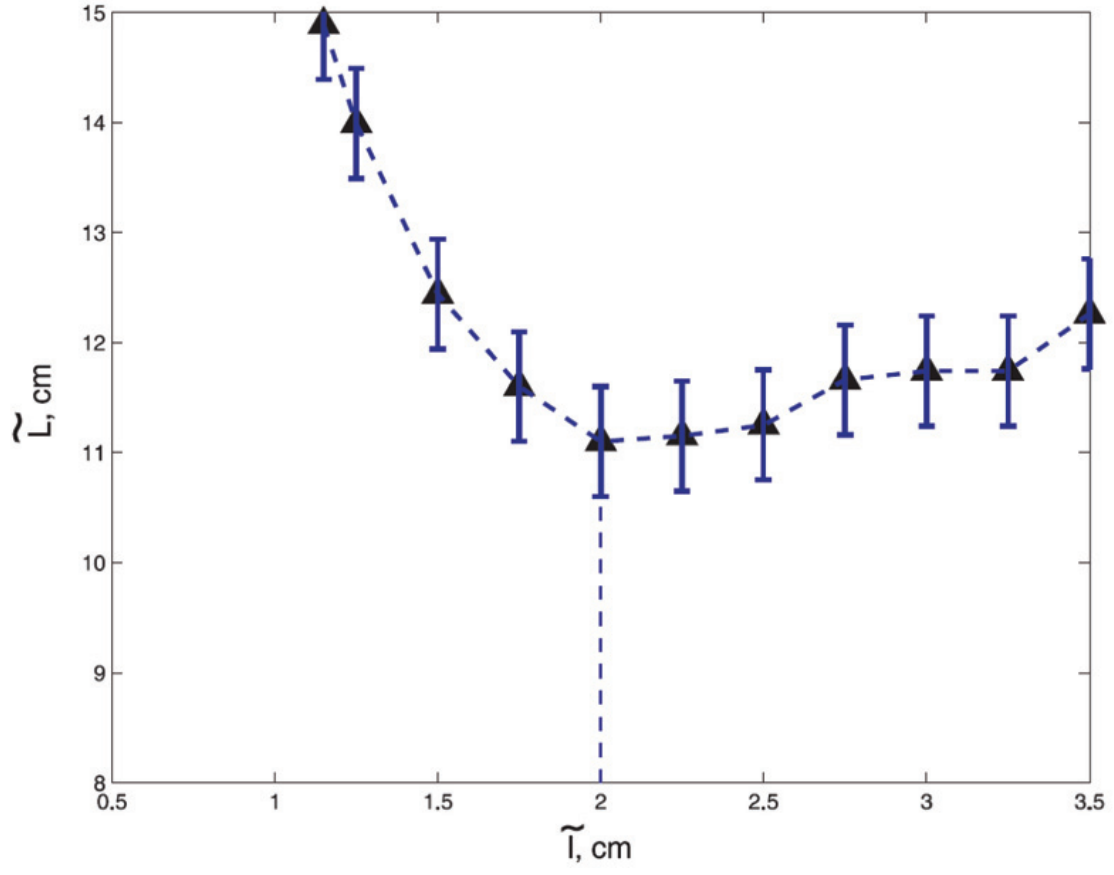


Figure 5: The locus of the 2D-3D transition for \tilde{L} as a function of the length of the transverse perturbation \tilde{l} for $Re = 10$. There is a pronounced minimum at $\tilde{l} = 2$ cm.

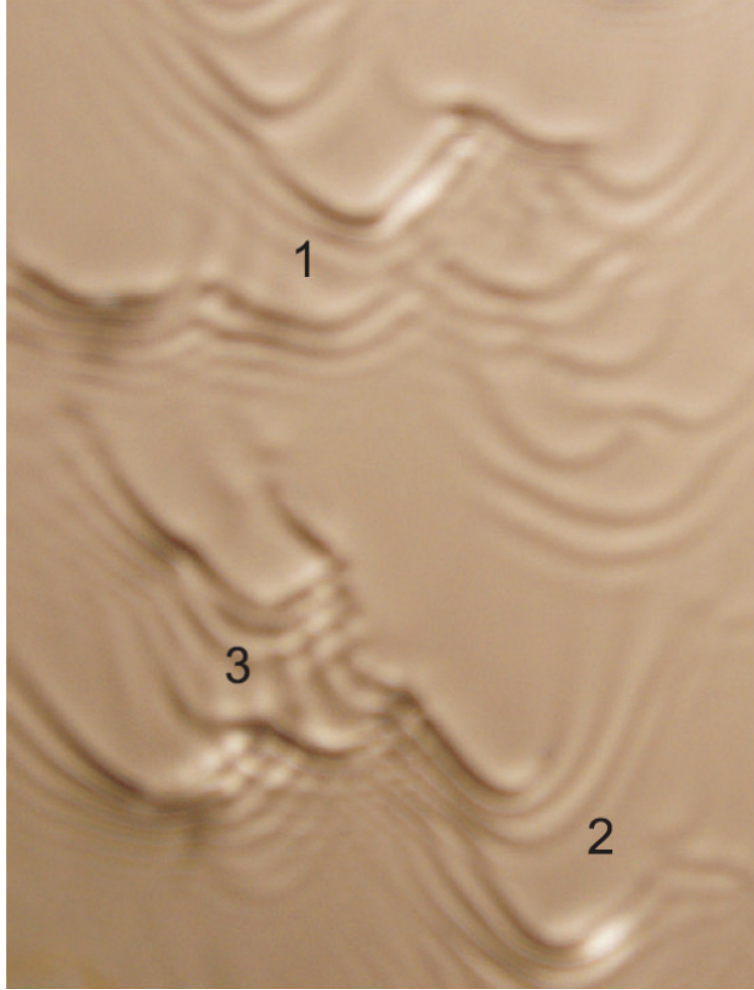


Figure 6: Typical example of the chaotic wave motion observed for $\langle Re \rangle = 60$. This section is located between 14 cm and 24 cm from the inlet. There are several typical wave patterns: 1 - 2D soliton with 3D modulation ready to be destroyed downstream; 2 - Well-developed 3D solitons interacting through their tails; 3 - Complex collision of several 3D solitons.

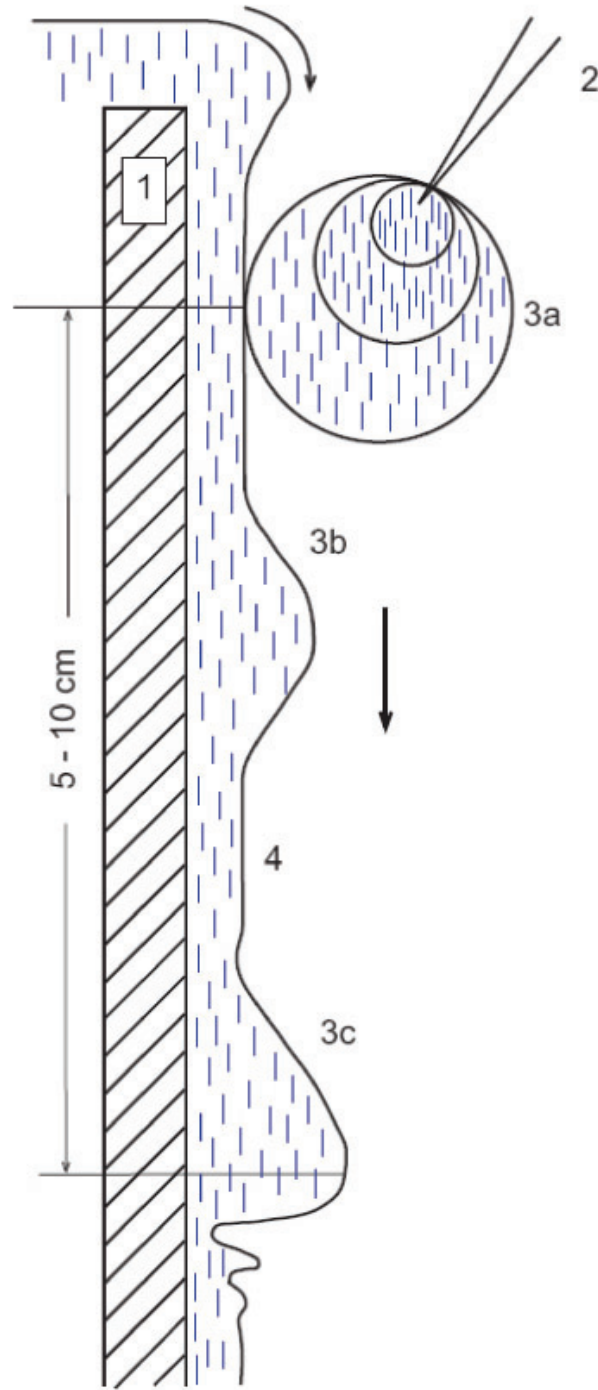


Figure 7: “Rain-drop” method of exciting 3D soliton with the mass of liquid in the pipette corresponding to the equilibrium 3D soliton mass. 1 - Test glass channel; 2 - Pipette with adjustable drop-volume; 3 - Successive instants of drop evolution, namely, 3a - drop is gradually increasing in size and finally touches the liquid surface and becomes involved in film flow, 3b - drop spreads over and turns into an evolving localized signal downstream which eventually 3c - becomes a stationary running Λ wave; 4 - liquid layer flowing down the channel.

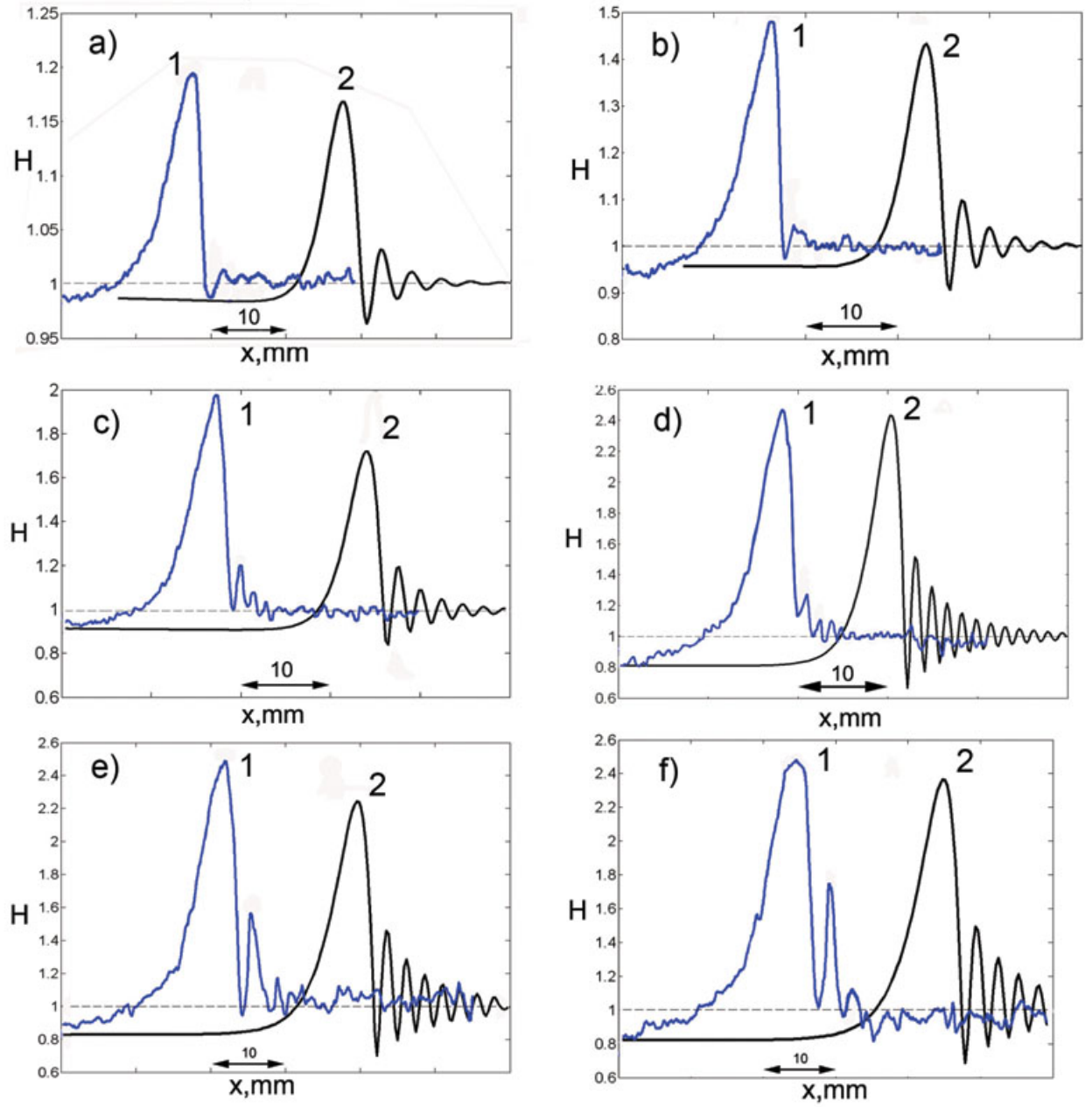


Figure 8: Experimental (1) and theoretical (2) cross sections, (a) $Re = 7$, (b) $Re = 8.8$, (c) $Re = 9.7$, (d) $Re = 13.2$, (e) $Re = 15$, (f) $Re = 17.7$.

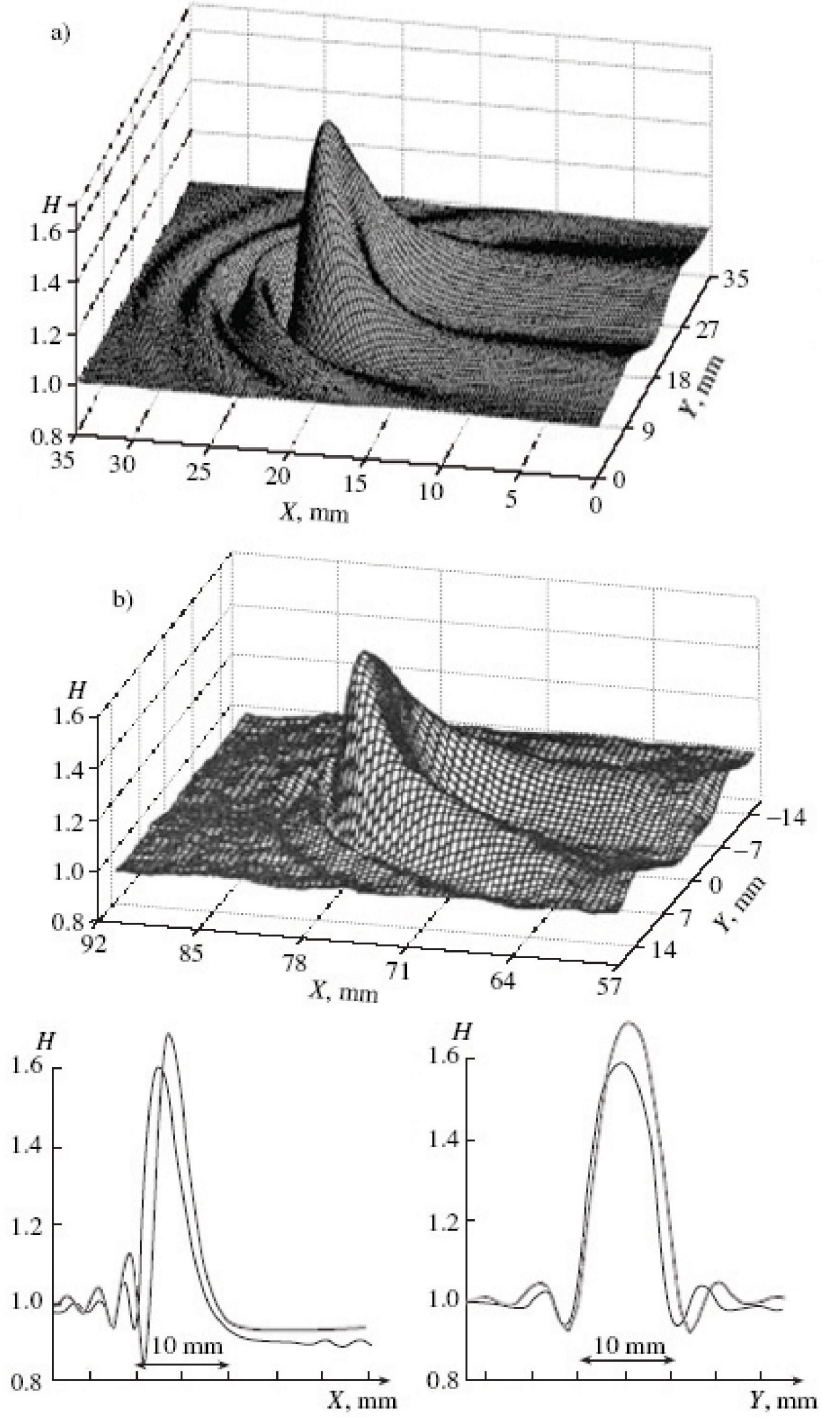


Figure 9: Comparison of our theoretical (a) and experimental (b) (see Alekseenko et al. [5], Fig. 4) profiles of Λ solitons and their cross sections for $Re = 3.9$, $\delta = 0.061$, $\gamma = 404$. Experimental wave velocity is 209 mm/s and theoretical velocity is 213 mm/s .

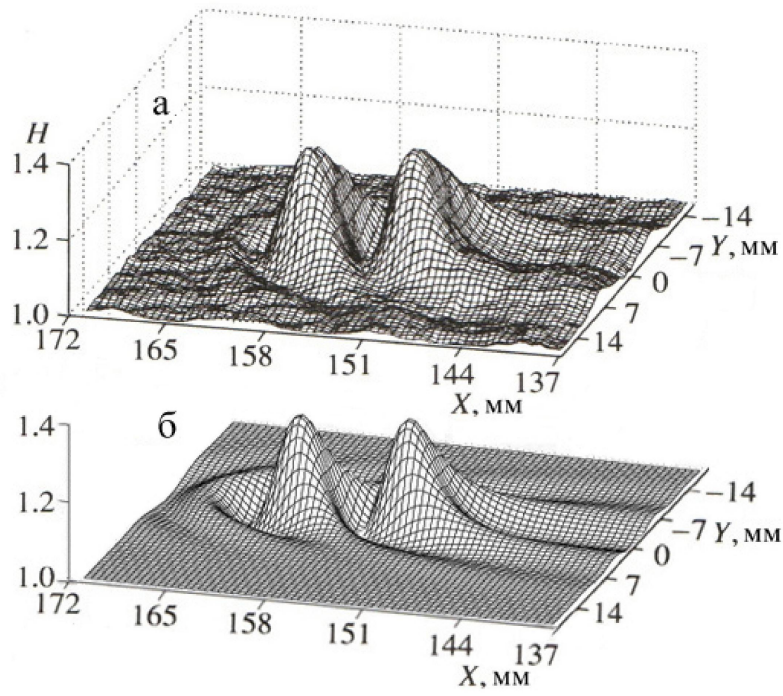


Figure 10: Comparison of our theoretical (a) and experimental (b)(see Alekseenko et al. [4], Fig. 3) profiles of two-hump 3D solitons for $Re = 2.2$, $\delta = 0.03$, $\gamma = 404$. Experimental wave velocity is 102 mm/s and theoretical velocity is 113 mm/s .

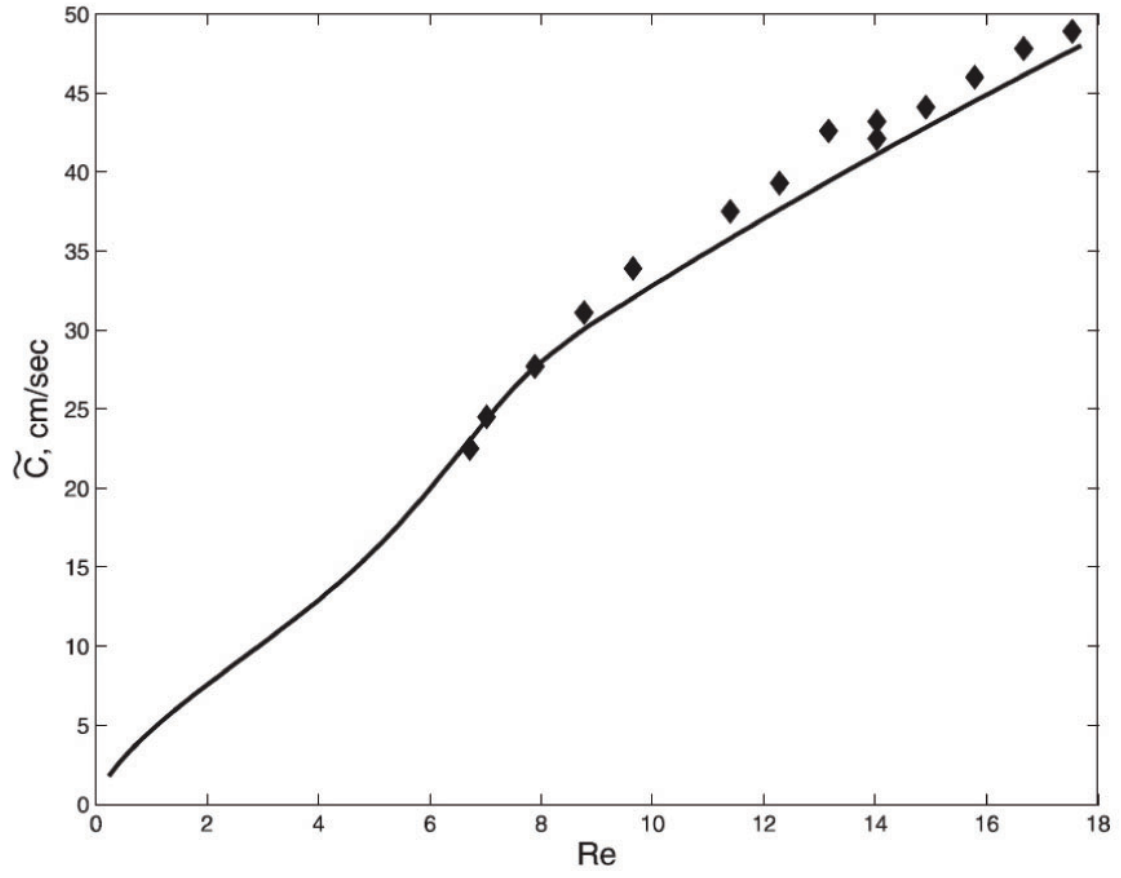


Figure 11: Phase velocities of Λ -solitons as a function of Re . The diamonds represent our experimental results.

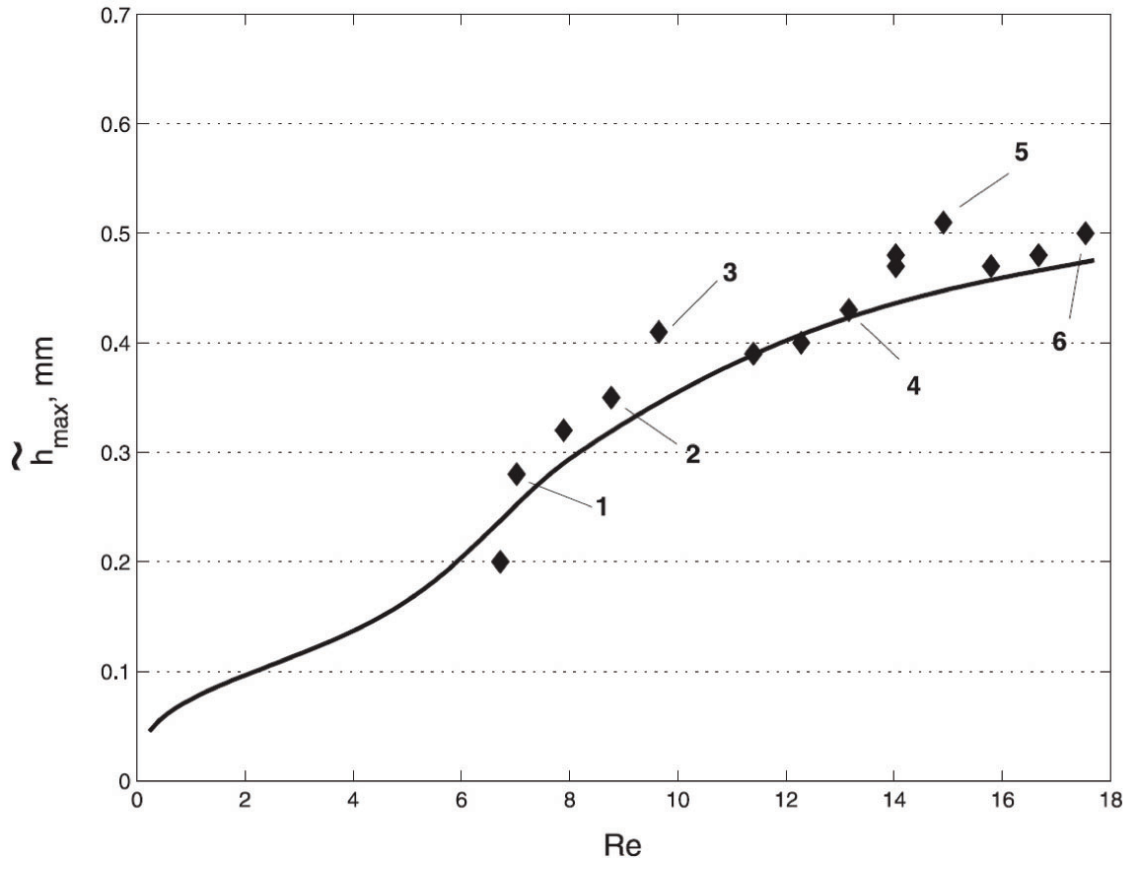


Figure 12: Maximum amplitudes of Λ -solitons as a function of Re . The diamonds represent our experimental results.

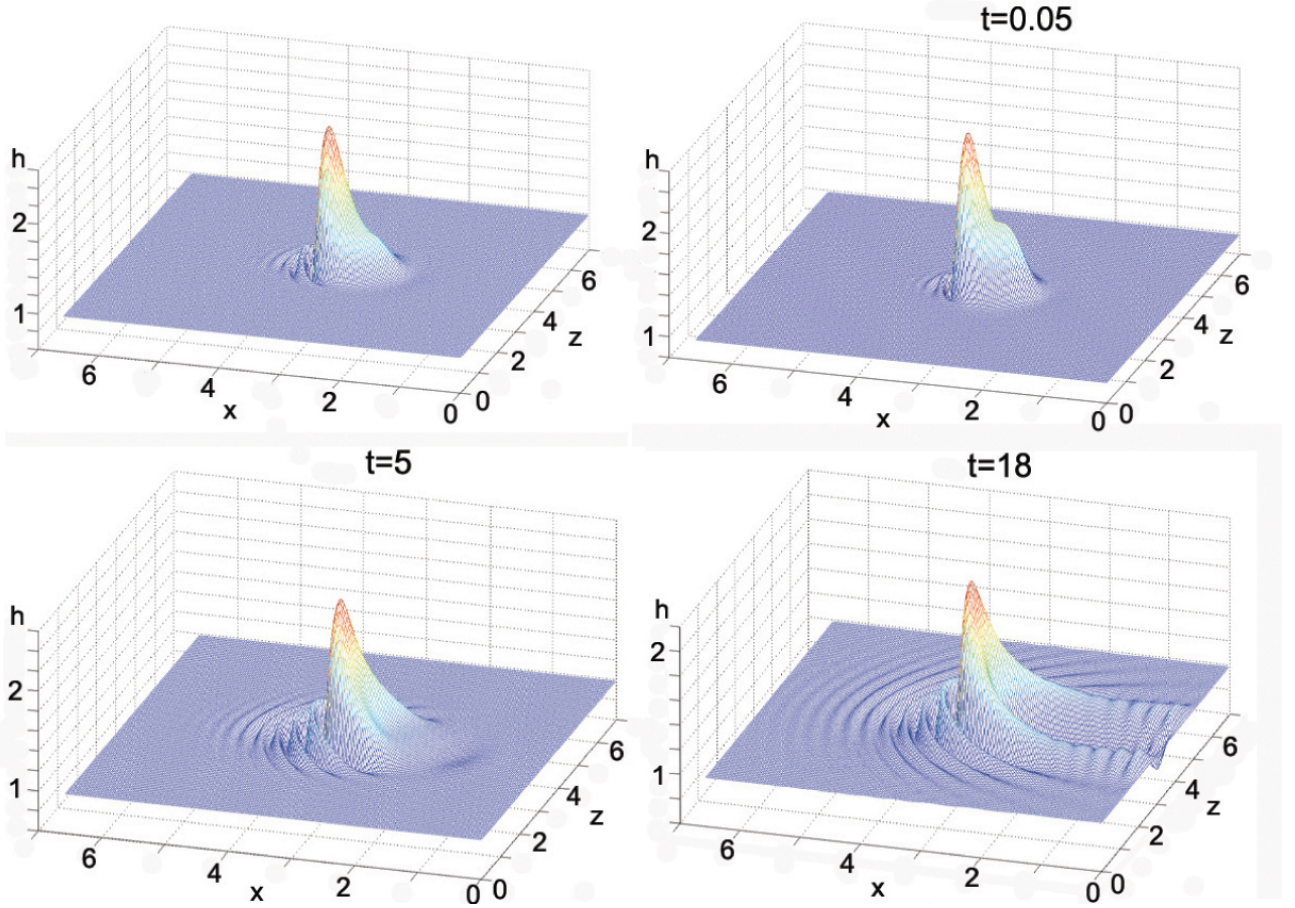


Figure 13: Formation of a Λ soliton from a localized signal, or “drop”, for $\delta = 0.0908$ at different times. At $t = 0.05$ the drop has a tendency to disintegrate. At $t = 1.0$ the capillary ripples form (not shown). At $t = 5.0$ the trough behind the head forms. At $t = 12.0$ we have reached a nearly steady 3D coherent structure. Finally, at $t = 18.0$ the 3D structure takes the shape of a Λ -soliton.

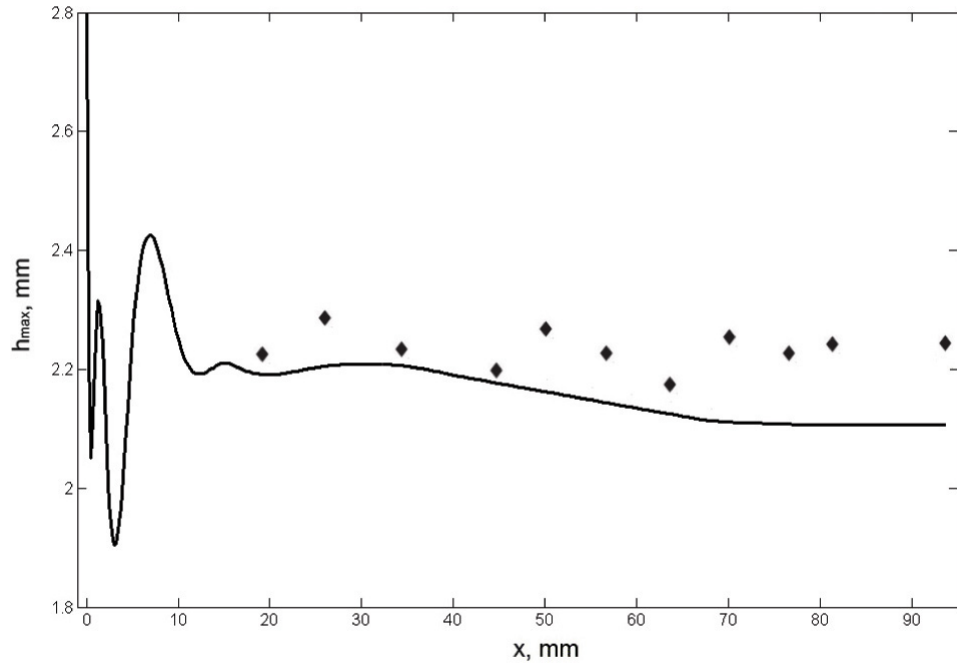


Figure 14: Evolution of the dimensionless amplitude h_{max} downstream, $\delta = 0.0908$, $Re = 9.28$ (water). The line stands for the calculations, the diamonds for our experimental results.

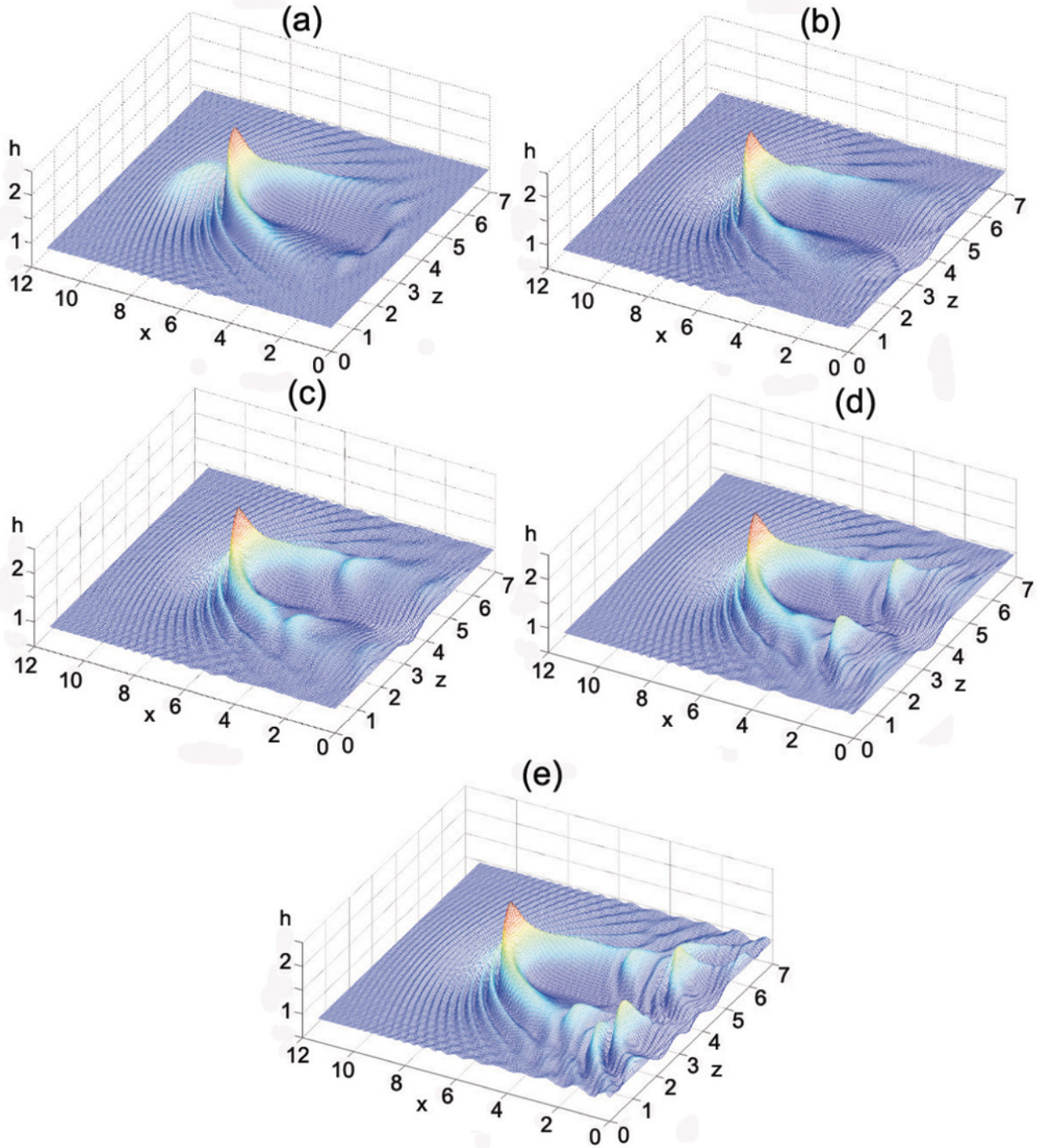


Figure 15: Coalescence of a Λ -wave with a localized signal of smaller amplitude and mass for $\delta = 0.15$, (a) $t = 0$ - 3D-soliton with a localized signal in front of it; (b) $t = 25$ - the mass is absorbed by the 3D-soliton which becomes an accelerated (excited) 3D-soliton; it is ready to swallow up an equilibrium soliton; (c) $t = 33$ - the extra-mass drains to the back of the soliton along its “moustaches”; (d) $t = 41$ - two small Λ -waves created from the extra mass; (e) $t = 45$ - “checkerboard” pattern starts to form.